

# Problems

Ted Eisenberg, Section Editor

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This section of the Journal offers readers an opportunity to exchange interesting mathematical problems and solutions. Please send them to Ted Eisenberg, Department of Mathematics, Ben-Gurion University, Beer-Sheva, Israel or fax to: 972-86-477-648. Questions concerning proposals and/or solutions can be sent e-mail to <eisenbt@013.net>. Solutions to previously stated problems can be seen at <<http://www.ssma.org/publications>>.

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*Solutions to the problems stated in this issue should be posted before  
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- **5385:** *Proposed by Kenneth Korbin, New York, NY*

A triangle with integer length sides and integer area has perimeter  $P = 6^6$ . Find the sides of the triangle when the area is minimum.

- **5386:** *Proposed by Michael Brozinsky, Central Islip, NY.*

Determine whether or not there exist nonzero constants  $a$  and  $b$  such that the conic whose polar equation is

$$r = \sqrt{\frac{a}{\sin(2\theta) - b \cos(2\theta)}}$$

has a rational eccentricity.

- **5387:** *Proposed by Arkady Alt, San Jose, CA*

Let  $D := \{(x, y) \mid x, y \in \mathbb{R}_+, x \neq y \text{ and } x^y = y^x\}$ . (Obviously  $x \neq 1$  and  $y \neq 1$ ).

Find  $\sup_{(x,y) \in D} \left( \frac{x^{-1} + y^{-1}}{2} \right)^{-1}$

- **5388:** *Proposed by Jiglaŭ Vasile, Arad, Romania*

Let  $ABCD$  be a cyclic quadrilateral,  $R$  and  $r$  its exradius and inradius respectively, and  $a, b, c, d$  its side lengths (where  $a$  and  $c$  are opposite sides.) Prove that

$$\frac{R^2}{r^2} \geq \frac{a^2 c^2}{b^2 d^2} + \frac{b^2 d^2}{a^2 c^2}.$$

- **5389:** *Proposed by José Luis Díaz-Barrero, Barcelona Tech, Barcelona, Spain*

Let  $ABC$  be a scalene triangle with semi-perimeter  $s$  and area  $\mathcal{A}$ . Prove that

$$\frac{3a + 2s}{a(a - b)(a - c)} + \frac{3b + 2s}{b(b - a)(b - c)} + \frac{3c + 2s}{c(c - a)(c - b)} < \frac{3\sqrt{3}}{4\mathcal{A}}.$$